

# Self-similar solutions as large-time asymptotics for space-fractional Stefan problems

Sabrina Roscani and Piotr Rybka

We investigate the large-time behavior of the one-phase fractional Stefan problem with a Caputo flux defined by

$$q(x, t) = -k_\alpha {}_0^C D^\alpha u(x, t) = {}_0 I^{1-\alpha} [u(\cdot, t)](x) = \frac{1}{\Gamma(1-\alpha)} \int_0^x u_x(z, t) (x-z)^{-\alpha} dz.$$

Motivated by the general principle that the long-time behavior of evolution problems is governed by steady states or special solutions such as self-similar solutions, we prove that the solution converges, as  $t \rightarrow \infty$ , to the self-similar solution obtained in [2] for the one-phase space-fractional Stefan problem with a fractional Neumann-type boundary condition at the fixed face,

$$-{}_0^C D_x^\alpha u(0, t) = \frac{h_0}{t^{\alpha/(1+\alpha)}}$$

The present work extends the corresponding result for the classical Stefan problem established in [1], where a variational approach was developed to analyze the large-time asymptotic behavior.

The fractional extension presented here not only demonstrates that the variational technique remains applicable in the nonlocal setting, but also incorporates several recent advances on Stefan problems with Caputo fluxes, including comparison and maximum principles, monotonicity of the free boundary, and regularity results in fractional Sobolev spaces.

## References

- [1] D. Hilhorst, S.Roscani, P.Rybka, *Convergence of solutions of a one-phase Stefan problem with Neumann boundary data to a self-similar profile*, *Nonlinear Differ. Equ. Appl.* **31** 56, (2024).
- [2] Roscani S. and Venturato L. *The similarity method and explicit solutions for the fractional space one-phase Stefan problems*, *Fractional Calculus and Applied Analysis*, Vol. 25, pp. 995–1021, 2022.

**First Author:** Sabrina Roscani  
**Affiliation:** *CONICET - Universidad Austral*  
*2000, Argentina*  
**e-mail:** sroscani@austral.edu.ar

**Second Author:** Piotr Rybka  
**Affiliation:** *Department of Mathematics, University of Warsaw*  
*Postcode, Poland*  
**e-mail:** rybka@mimuw.edu.pl